

# A Novel Energy Minimization Criterion for Color Image Segmentation \*

Adolfo Martínez-Usó      Filiberto Pla      Pedro García-Sevilla  
Departamento de Lenguajes y Sistemas Informáticos  
Universitat Jaume I, 12071 Castellón (Spain)  
{auso, pla, pgarcia}@lsi.uji.es

## Abstract

*In this article, we present an unsupervised segmentation algorithm through a multiresolution approach which uses both color and edge information with a quadtree structure, through as well as an iterative minimization process of an energy function. The algorithm has been applied to fruit images in order to distinguish the different areas of the fruit surface in fruit quality assessment applications. Due to the unsupervised nature of the method, it can adapt itself to the huge variability of colors and shapes of the regions in fruit inspection tasks.*

## 1. Introduction

Many segmentation techniques are available in the literature, and some of them have been widely used in different application problems. These techniques mainly differ in the criterion used to measure the similarity of two regions and in the strategy applied to guide the segmentation process. In our algorithm, we have paid great importance to an efficient strategy to solve the problem. The work we present here develops a robust criterion by combining color and edge information and an accurate strategy as the basis for the whole segmentation process that guides all the algorithm steps.

Our main motivation has been to obtain a method able to segment images of fruits for their quality classification. This type of application implies the following requirements: (1) An *unsupervised method* would be needed due to manifold variables which can arise in the fruit images. Thus, any prior knowledge should be avoided for the segmentation procedure. (2) The segmentation method has to be mainly based on *color and edge criteria*, in order to define the segmented region boundaries as accurately as possible.

To meet the above mentioned requirements, a multiresolution Quadtree (QT) structure has been chosen to support the developed method due to its computational efficiency. Image segmentation will be conditioned and orientated by

the image representation adopted, that is, by the QT and by the color and edge information as a particular criterion.

As part of any segmentation process, a criterion or condition is needed to know when the final segmentation has been reached. In our algorithm, the *ideal segmentation state* is achieved through a variational method that minimizes the segmentation energy.

The main motivation contribution of the presented work is the proposed energy function that efficiently combines color (intra-region features) with edges (borders information) using a computationally efficient hierarchical representation, a QT, to guide the minimization process. The proposed framework yields satisfactory results, particularly in fruit inspection tasks.

## 2. Variational Image Segmentation

The goal of variational methods for image segmentation is to develop algorithms and their mathematical analysis to minimize the segmentation energy  $E$  represented by a real value. This energy can also be regarded as a measure of the segmentation state. It is a general agreement that the smaller value of  $E$ , the better the segmentation. The segmentation energy measures how smooth the regions are, the similarity between the segmented image and the original one and the similarity between the obtained edges and the discontinuities of the original image.

The Mumford-Shah model [5] has been regarded as a general model within variational segmentation methods. This model looks for a piecewise smoothed image  $u$  with a set of discontinuities, edges of the original image  $g$ .

According to Mumford-Shah's conjecture, the minimal segmentation exists but it is not unique; for each image a set of minimal segmentations exists. Therefore, with the aim of minimizing the segmentation energy we can minimize the following equation, where  $K$  is the set of discontinuities in the image domain  $\Omega$  representing the *edges* of  $g$ :

$$E(u, K) = \int_{\frac{\Omega}{K}} (|\nabla u(x)|^2 + (u - g)^2) dx + \text{length}(K) \quad (1)$$

\* This work has been partly supported by grants DPI2001-2956-C02-02 from Spanish CICYT and IST-2001-37306 from the European Union

Since Mumford-Shah's work, several approaches appeared that suggested modifications to the original scheme. Recent works change equation (1) in order to improve the results. In this sense, the boundary function, which is binary in the Mumford and Shah's formulation, was changed by a continuous one which obtains a clearly defined boundary in [4]. Furthermore, in [2] the authors analyze some possible generalizations of the Mumford-Shah functional for color images. They suggest that these changes accentuate different features in edge detection and restoration.

In general, formulating variational methods have several advantages:

1. A variational approach returns explicitly a measure of the quality of the segmentation. Therefore, we are able to know how good the segmentation is.
2. Many segmentation techniques can be formulated as a variational method.
3. A variational approach can be used as a quantitative criterion in order to measure the segmentation quality.
4. Finally, a variational approach provides a way to implement non-supervised processes by looking for a minimum in the segmentation energy.

### 3. Energy Minimization of the Quadtree Image Representation

In this work, a global function to minimize the segmentation energy is proposed. Therefore, the functional takes into account the whole image energy in order to decide about continuing the minimization process. With this assumption, it is important to point out that we cannot guarantee to find a global minimum but, the experimental results obtained show that the solutions are very satisfactory.

Let  $u$  be a smoothed image and a set of discontinuities of the original image  $g$ , let  $R_i$  be a set with  $0 < i \leq \|\Omega\|$  and  $R_i \subseteq \Omega$ .  $R_i$  is a family of  $r$  regions in  $\Omega$  such that  $\bigcup_{i=1}^r R_i = \Omega$  and  $R_i \cap R_j = \emptyset$  for  $i \neq j$ . Let  $B_i$  represent the border of region  $R_i$ , that is,  $R_i - B_i$  is the interior of region  $R_i$ .

Thus,  $\forall x \in R_i$  let us consider the function  $E_i(R_i, B_i)$ :

$$E_i(R_i, B_i) = \int_{R_i} (|u(x) - m_i|) dx + \frac{\int_{R_i - B_i} |\nabla g(x)|}{\int_{B_i} |\nabla g(x)| + 1} dx + \|\Omega\| \quad (2)$$

In the image  $u$ ,  $u(x)$  returns the color of an  $R_i$  element and  $m_i$  is the average color value of  $R_i$ . The final segmentation energy is expressed as  $E(\Omega) = \sum_i E_i(R_i, B_i)$ .

The QT structure allows us to divide an image within a complete multiresolution tree representation including neighboring information. This spatial information can be

further used by a merging strategy which joins the QT leaves using color and edge information.

Let us consider the following discrete version of (2) with the same nomenclature,

$$E_i(g) = k \cdot H_i + (1 - k) \cdot G_i + \sqrt{r} \quad (3)$$

that returns the segmentation energy at each iteration,  $H_i$  and  $G_i$  are terms as follows:

$$H_i = \frac{\sum_{R_i} (D(u(x), m_i))}{\|R_i\|} \quad (4)$$

$$G_i = \frac{\sum_{R_i - B_i} |\nabla g(x)|}{\sum_{B_i} |\nabla g(x)| + 1} \quad (5)$$

Specifically, in the QT representation,  $R_i$  is the set of leaves of the QT belonging to region  $i$  and  $B_i$  represents the boundary leaves in  $R_i$  with  $0 < i \leq r$  where  $r$  is the number of regions at each iteration. The function  $D$  calculates the distance between two colors. The value  $\|\nabla g(x)\|$  returns the gradient magnitude in the pixel  $x$ . Note that the parameter  $0 < k < 1$  allows to define the weight between the color and boundary information.

Thus, in the energy functional (3) we can distinguish three different components:

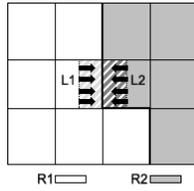
1. The first one takes into account the homogeneity of each region by means of a similarity measure from each pixel to its region average value.
2. The second component promotes that the gradient magnitude will be low in the interior leaves and high in the boundary ones.
3. Finally, the third term has been inspired by the measure of the segmentation quality in [1]. It helps the function to punish a large number of regions and plays the same role as the boundary length in many approaches about variational image segmentation [4][2].

Throughout the segmentation process, the minimized segmentation energy involves the minimized QT representation of the image by pruning the QT. By means of the pruning process of the spatial representation, a tree with less levels and less regions by level is achieved at each iteration.

## 4. The Algorithm

### 4.1. Use of Color Information

As a color discriminant measure, we have used an Euclidean distance in the L\*a\*b\* space in order to calculate the distance between two colors. This assumption is taken because in L\*a\*b\* the distances between two colors are approximately proportional to the human perceptual difference between them, and it appears to possess more uniform perceptual properties than others CIE spaces [3].



**Figure 1. Representation of the edge criterion**

In this work, an internal process (inner local loop in the iterative process) has been implemented in order to consider whether two regions must be merged or not (see Sect. 4.3). In this internal process, besides the color criterion of the functional (3), we have another color criterion that can be summarized in two steps:

1. The luminance of each region is considered as a set of pixels which follow a Normal distribution. So, the average value  $\mu$  and standard deviation  $\sigma$  in  $L^*$  are calculated for each region. The probability density function of each Normal distribution is computed and these distributions are compared according to the absolute value of their differences,  $|N(\mu_A, \sigma_A^2) - N(\mu_B, \sigma_B^2)|$ .
2. The chroma of each region ( $a^*b^*$ ) is taken into account and the Euclidean distance between the average values of each region is used to measure the color difference between both regions.

Finally, it is important to emphasize that we have decided to take into account the intensity plane  $L^*$  and the chroma planes  $a^*b^*$ . Therefore, our results involve a special sensitivity to brightness and shadows. However, we can put into practice the same strategy only with the chroma information to avoid the intensity segmentation or other invariant color representations to some of the color components [6].

## 4.2. Use of Edge Information

The use of edge information in the segmentation process tries to avoid the merging of regions when the color criterion is satisfied but there exists an edge between the regions. In this sense, gradient information is used to develop a boundary map which is checked as the edge criterion.

Let  $R1$  and  $R2$  be two regions with a common boundary and let  $L1$  and  $L2$  be two leaves of the QT that belong to regions  $R1$  and  $R2$  respectively as shown in Fig. 1.  $L1$  and  $L2$  share a linear segment of the common boundary between the two regions. From the leaves  $L1$  and  $L2$  we can compute the values  $\sum_{half} \nabla_1$  and  $\sum_{half} \nabla_2$  which correspond to the sum of half of the values of the gradient magnitude in the perpendicular direction of the edge in the corresponding leaf (striped patterns in Fig. 1). For each leaf, the gradient

values close to the boundary segment are taken into account. In this way, equation  $FB = (\sum_{half} \nabla_1 + \sum_{half} \nabla_2) / \|S\|$  is computed for each boundary segment where the denominator represents the segment size.

After all boundary segments have been processed, we compute  $\frac{\sum(FB)}{N}$  where  $N$  is the number of segments that form the whole boundary between the regions. In the inner loop (Sect. 4.3) if the final value obtained does not exceed a fixed threshold, the two regions could be merged into one.

## 4.3. The Segmentation Process

The whole segmentation process can be summarized in the following algorithm:

1. Perform the Principal Component Analysis of the input image to construct the gradient magnitude image from the first component by means of a Sobel filter of the original smoothed Gaussian image.
2. Construct an oversegmented representation of the  $L^*a^*b^*$  image, that is, expand the QT until every square region have all pixels with the same color. After this, create an ordered list according to region sizes and calculate the whole image energy using (3).
3. *Internal Process*: Color homogeneity is tested for each region and its neighbor regions. At the same time, the algorithm looks for border leaves between regions and checks whether there are high gradient values. If both criteria are fulfilled, then the regions are merged.
4. Finally, the algorithm computes the functional (3) again. If the functional has decreased, reorder the list of clusters and repeat step 3 until the functional became stable or worse (iterative sequence that may be seen as a coarse-to-fine segmentation process).

It is important to point out that during the merging process regions are arranged according to their size, giving more importance to bigger regions and making easier merging small regions with big ones. In addition, only regions with a common boundary can be merged. This represents the spatial constraint in the merging process.

Fig. 2 shows the outline of the segmentation strategy used, which is basically an iterative process representing an *internal process* where each cluster is compared to all its neighboring clusters and it will be merged when the segmentation criterion is satisfied (see Sect. 4.1, 4.2). By this inner loop, the process avoids getting trapped in little local minima of the energy function. This clustering process stops when no other merging can be performed without increasing the energy of the segmentation. Finally, small re-

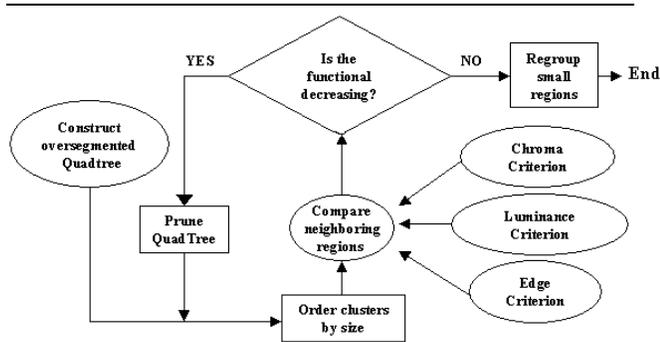


Figure 2. Segmentation process flow chart

gions are ignored and merged to their most similar neighbors giving more importance to the biggest ones.

## 5. Results

Results obtained with some classical color images (Fig. 3) as well as with fruit images (Fig. 4) are presented. First column represents the original images, the second one the segmentation images, and the last one shows the edge results, where the darker the edge line is, the greater difference between the color of neighboring regions.

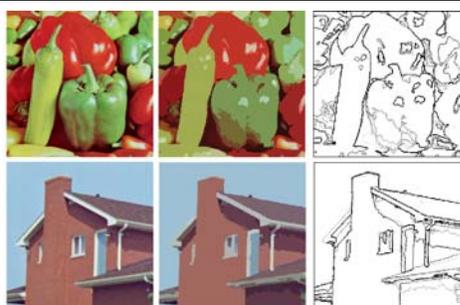


Figure 3. Real images segmentation.

Fruit image segmentation is used as input in a further process to characterize and classify the fruit surface. These visual inspection applications identify and detect different types of defects and parts of the fruit. In this sense, images in Fig. 4 show some examples of the segmentation results on different fruits and situations to be characterized. Note how the segmentation process has adapted to the regions of each image due to its unsupervised nature. For instance, the first column shows examples of fruits with various stains produced by the effect of rot and how the segmentation obtained has found the different variations of the stains of the rotten zone. This will allow the extraction of region descriptors for their identification and classification.

## 6. Conclusions

In this paper, an unsupervised color segmentation algorithm has been presented. The results obtained show how the algorithm can adapt to the different situations and variability of color regions, being able to segment areas and locating the borders due to the use of gradient information during the segmentation process.

Color and edge information combined with a QT representation and the minimization function presented is the base of an unsupervised segmentation strategy that can locate color regions, and find their contours satisfactorily.

Further work is directed to test the present work in other application problems, and to improve the efficiency of the algorithm in both, the quality of the segmentation and the computational burden, using some spatial diffusion techniques combined with the variational criterion proposed and the QT-based multiresolution approach.

## References

- [1] M. Borsotti, P. Campadelli, and R. Schettini. Quantitative evaluation of color image segmentation results. *Pattern Recognition Letters*, 19:741–747, 1998.
- [2] A. Brook, R. Kimmel, and N. Sochen. Variational restoration and edge detection for color images. *Journal of Mathematical Imaging and Vision*, 18(3):247–268, 2003.
- [3] H.-D. Cheng and Y. Sun. A hierarchical approach to color image segmentation using homogeneity. *IEEE Transactions on Image Processing*, 9(12):2071–2082, 2000.
- [4] G. Hewer, C. Kenney, and B. Manjunath. Variational image segmentation using boundary functions. *IEEE Transactions on Image Processing*, 7(9):1269–1282, 1998.
- [5] D. Mumford and J. Shah. Optimal approximations by piecewise smooth functions and associated variational problems. *Comm. in Pure and Applied Mathematics*, 42(4), 1989.
- [6] J.-M. G. Rein van den Boogaard, A. W. Smeulders, and H. Geerts. Color invariance. *IEEE Transactions on PAMI*, 23(12):1338–1350, December 2001.

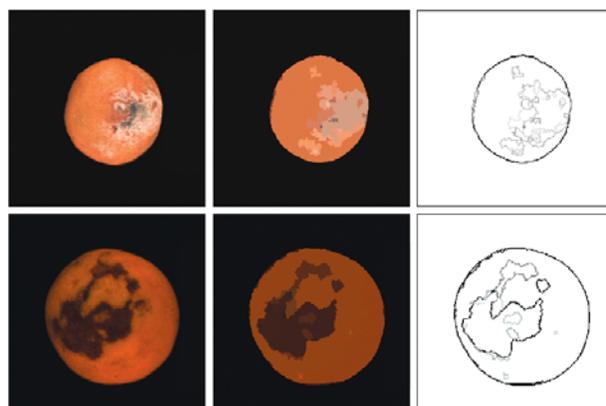


Figure 4. Fruit segmentation results.