Classification of Binary Textures Using the 1-D Boolean Model

Pedro García-Sevilla and Maria Petrou

Abstract—**The one-dimensional (1-D) Boolean model is used to calculate features for the description of binary textures. Each two-dimensional (2-D) texture is converted into several 1-D strings by scanning it according to raster vertical, horizontal or Hilbert sequences. Several different probability distributions for the segment lengths created this way are used to model their distribution. Therefore, each texture is described by a set of Boolean models. Classification is performed by calculating the overlapping probability between corresponding models. The method is evaluated with the help of 32 different binary textures, and the pros and cons of the approach are discussed.**

Index Terms—**Feature extraction, statistical models, texture analysis.**

I. Introduction

In this work, we use the Boolean model for texture modeling. This model has been investigated intensively from the theoretical point of view [1], [3], [4], [7], [9], but whether it could be used for real texture images in practice has not been established yet.

The simplest form of the Boolean model is the one-dimensional (1-D) version of it [3], [4], [9]. As texture is a two-dimensional (2-D) spatial property, in order to bypass the problem of dimensionality, and introduce some spatial property to the 1-D model, we use it in conjunction with horizontal, vertical, and Hilbert scannings of the image.

The Boolean model consists of two independent statistical processes, a shape process and a point process [1]. The outcomes of the shape process determine the shapes of the primitives, while the outcomes of the point process determine where these shapes appear. In a typical realization of a Boolean model, shapes tend to overlap each other.

In the 1-D case, the shapes of the Boolean model are simply line segments. The locations of the origins of the shapes are the outcomes of a point process with probability p, the *marking probability*. The lengths of the line segments are distributed according to some distribution function $C(k)$, where $k = 1, 2, \cdots$. Thus, $C(k)$ is the probability that a line segment has length less than or equal to k, where k is discrete and $C(0) \equiv 0$ [1].

II. Parameter Estimation

Dougherty and Handley have shown that for a given observation, we can have an estimate of parameter p, called $\tilde{p}$, as the inverse of the mean length of uncovered runlengths, that is, $\tilde{p} = \frac{1}{m}$ [1]. They also showed that the probability of observing a certain combination S' of runlengths is given by

$$P(S') = \prod_{l=1}^{m_x} P(L = l)^{\beta(l)} p^n (1-p)^{\omega - n} \tag{1}$$

where $P(L = l)$ is the probability of observing a runlength of size l, $\beta(l)$ is the number of covered runlengths of length l, m_x is the length of the maximum covered runlength, n is the number of uncovered runlengths, and ω is the total number of uncovered pixels.

This equation can be used to compute the probability of a sequence of runlengths, provided we construct the histogram of covered runlengths $\beta(l)$. Then we can compute the likelihood of observation Z for a given probability distribution $C(k)$ as

$$L(Z; C(k)) = (1-p)^{w-n} p^n \prod_{i=1}^{m_x} P(L = i)^{\beta(i)} \tag{2}$$

where $P(L = i)$ can be computed using $C(k)$ and the estimated $\tilde{p}$ value.

So, if we have a set of probability distributions and we want to choose one of them as an estimation of the distribution of the Boolean model, we compute the log likelihood L for each candidate distribution, and choose the one which maximizes it.

A. Influence of the Observation Size in the Estimation

In order to study the size of observation needed to obtain a good estimation of the parameters of the Boolean model, we generated 100 different random strings of 65 536 pixels each using a marking probability $p = 0.2$ and a shifted Poisson distribution (so that $C(0) = 0$) with $\theta = 4$ for the shape process. A series of experiments were contacted, according to which the parameters of the Boolean model were estimated for each string using the whole string, one half of it, a quarter of it, and so on. The set of functions used in the estimation of the probability distribution was formed by shifted Poisson distributions with parameter θ varying from one to ten with increments of 0.5. That is, the set contained a total of 20 probability distributions.

Fig. 1 shows the mean percentage error obtained over 100 strings for each observation size. The mean percentage error plus one standard deviation of the distribution of these errors over the 100 strings used is also plotted. The left panel refers to the marking probability, while the right one refers to parameter θ of the shifted Poisson distribution. It can be noticed that even for samples of only 256 pixels, the mean error is less than 20% for both parameters.

B. Influence of the Precision in the Estimation

We know that $P(L = m)$ tends to zero when m is large enough. We want to study the influence of truncating the computation of $P(L = m)$ to the estimation of the probability distribution used in the shape process.

Two different experiments were carried out. In the first one we generated 100 random strings of 4096 pixels using a model with $p = 0.2$ and for $C(k)$ a discrete Normal distribution DN1 defined as:

$$\mathrm{DN1}(k) = \begin{cases} 0 & k = 0 \\ \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{k-\mu}{\sigma\sqrt{2}}\right)\right) & k = 1, 2, \cdots \end{cases} \tag{3}$$

with $\mu = 4.0$ and $\sigma = 1.0$. In the second experiment, the parameters used were $p = 0.3$, $\mu = 6.0$, and $\sigma = 10.0$. In both cases, we estimated the original parameters, truncating the computation of

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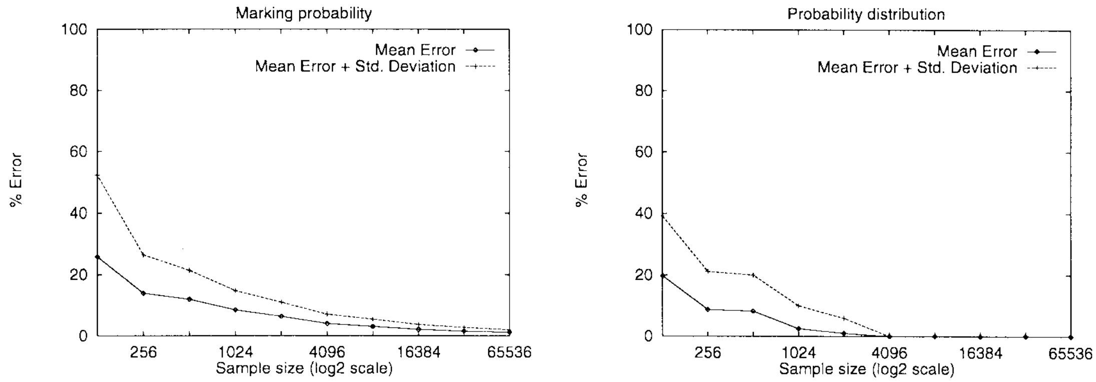


Fig. 1. Mean error obtained after 100 experiments for estimation of the marking probability (left) and parameter θ (right) as a function of the sample size.

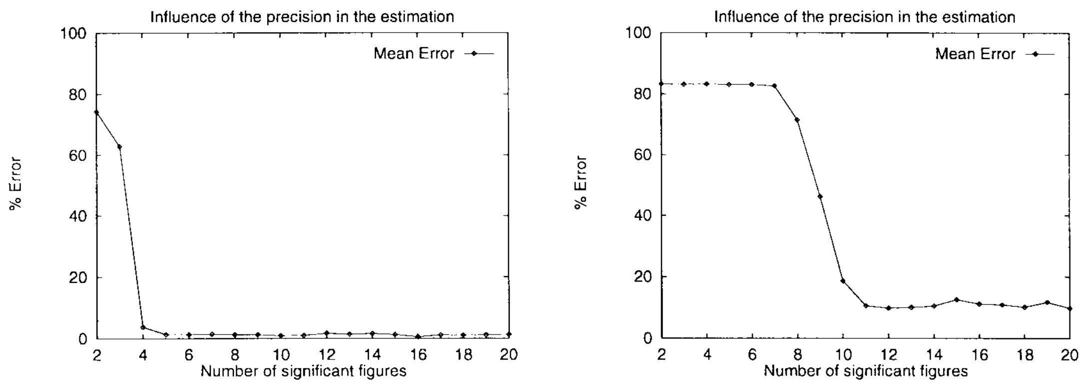


Fig. 2. Mean error obtained over 100 experiments in the estimation of parameter μ as a function of the number of significant figures retained in $P(L = m)$. The true parameter values are: Left: $p = 0.2,\ \mu = 4,\ \sigma = 1$. Right: $p = 0.3,\ \mu = 6,\ \sigma = 10$.

$P(L = m)$ at different levels of accuracy, varying from 10^{-2} to 10^{-20} (i.e., setting to zero values which were equal to or smaller than the specified accuracy).

The set of distributions tried in the estimation of the probability distribution was formed by discrete Normal distributions with parameter μ varying from 0.5 to ten with increments of 0.5 and parameter σ varying from one to ten with increments of one, that is, the set contained a total of 200 probability distributions.

Fig. 2 shows the results obtained for the first (left panel) and second (right panel) experiment. It can be noticed that for the first experiment only four or five significant figures are enough to yield a satisfactory estimation. However, the second experiment gave very poor results when using less than ten significant figures. This is due to the fact that in the second experiment we used a greater marking probability and the probability distribution used produced very long shapes. These may result in even longer covered runlengths. This means that the probability density $P(L = m)$ is spread, resulting in very small values. In this case, the truncation of the distribution $P(L = m)$ can influence significantly the parameter estimation process.

III. Classifying Real Textures

To enhance the variety of parameters that we estimate, we use the following.

1) Horizontal, vertical, and the four Hilbert scannings [8].
2) Direct and inverse video. By "direct video" we mean that the parameters are estimated considering white runlengths as being the covered runlengths of the Boolean model. By "inverse video" we mean that the white runlengths of the image are considered as the uncovered runlengths of the Boolean model. We refer to this as two *color assignments*.

The features we use for texture classification are the parameters of Boolean models. So, as a measure of similarity between two textures we must use a similarity measure between two Boolean models. Let p_1 and $C_1(k)$ characterize the first Boolean model and p_2 and $C_2(k)$ characterize the second Boolean model. To obtain an overall similarity measure between two Boolean models, we count as positive the similarity measure between their corresponding *probability density functions* (pdf's) and as negative the similarity measure between their marking probabilities.

$$S(p_1, C_1, p_2, C_2) \equiv \left[\sum_{k=1}^{\infty} \mathrm{Min}\,\{C_1(k) - C_1(k-1),\ C_2(k) - C_2(k-1)\}\right] - |p_1 - p_2|. \quad (4)$$

When $S = 0$, the two models either have identical distributions and totally different marking probabilities, or they have totally nonoverlapping distributions and identical marking probabilities (The function $1 - S$ is a distance metric between two models, consisting of the sum of two well known metrics, namely the integral metric for the difference of two distributions divided by two, plus the absolute difference metric, for the difference of two numbers).

The similarity function S takes values in the interval $[-1, 1]$, where 1 means that the two Boolean models are identical and -1 means that they are totally different.

When each texture in the database is described using only one of all possible Boolean models (the one that corresponds to the maximum likelihood and therefore describes it best), a test image

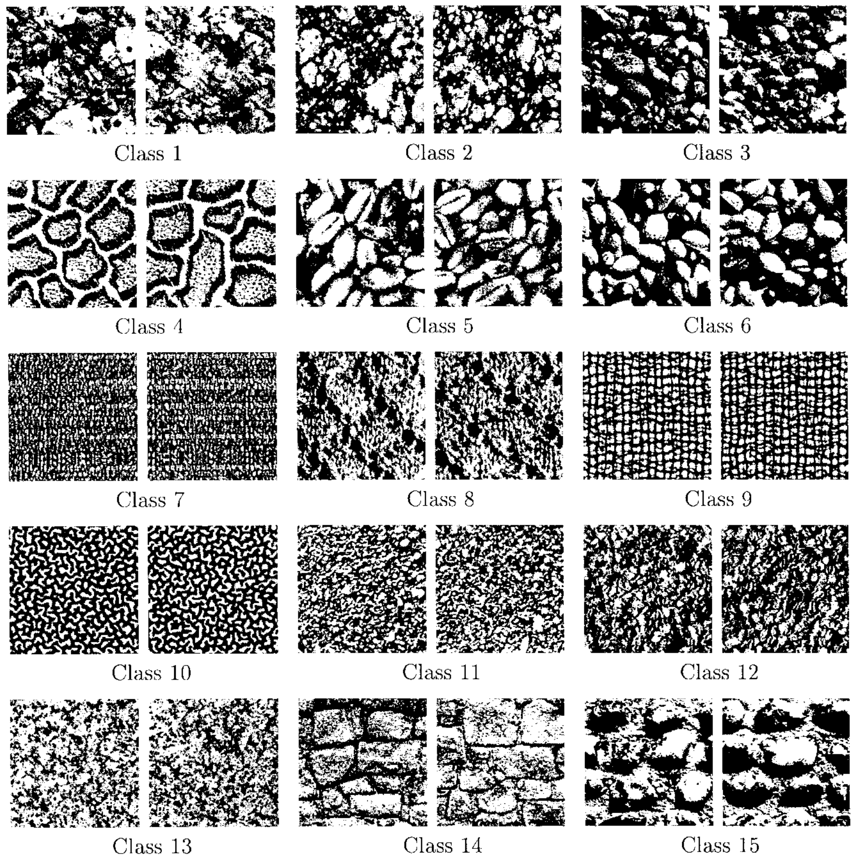


Fig. 3. Texture classes 1–15.

will be identified with the texture for which the similarity measure of (4) becomes maximum. Note that different images in the database may be described by different scannings and color assignments. Thus, all possible estimations may be necessary for the test image. We will refer to this approach of classification as the MAXL criterion.

When the textures in the database are described using all available Boolean models corresponding to different scannings and color assignments, a test image may be classified in the following two different ways.

1) Each Boolean model of the test image is compared with the corresponding Boolean model of the textures in the database, so that several similarity values between the test image and each entry in the database are obtained. The final similarity measure between the test image and a model image will be the average of the similarity measures over all the Boolean models. The test image will be identified with the model texture that maximizes this value. We will refer to this criterion of classification as the MEAN criterion.
2) The similarity measure between the test image and a model image is given by the *minimum* similarity of all the Boolean models considered. As before, the test image will be identified with the model that maximizes this measure. We will refer to this criterion as the MAXMIN criterion.

IV. Experiments

To check the performance of the above features, we chose 32 different binary textures digitised from [6] using 100 dpi resolution. The images were quite large and each was divided into two, one half used as model and the other half for testing. This allowed us to have two different images for each texture of size 256 × 256 pixels each. Figs. 3 and 4 show the texture images used in the experiments. The right panels are the models.

Further, a database of probability distribution functions was created. A family of Poisson distributions was included in this database with parameter θ varying from 0.25 to 25.0 using increments of 0.25. A family of Rayleigh distributions as well as Maxwell distributions were included in the database, both computed for the same range of parameters as the Poisson distribution [5]. Using the Normal distribution, two different families of distribution functions were included, $\mathrm{DN1}(x)$ and $\mathrm{DN2}(x)$, where $\mathrm{DN2}(x)$ is defined as

$$\mathrm{DN2}(k) = \frac{1}{\sum_{x=0}^{\infty} n(x)} \sum_{x=0}^{k} n(x) \quad \text{for } k = 0, 1, 2, \cdots \qquad (5)$$

where $n(x)$ is the normal density function.

In this case, parameter μ varied from 0.2 to 25.0 using increments of 0.2, while parameters σ varied from 0.5 to 25.0 using increments of 0.5. Also sets of Gamma and Beta distribution functions were included in the database, computed for the same range of parameters [5]. The Beta distribution functions were scaled to several intervals. After some preliminary tests, only one family of Beta distributions scaled to the interval $[0, 50]$ was retained in the database, because it seemed to perform better than Beta distributions scaled to other intervals. A total of 25 300 different distributions were used.

Different experiments were carried out with and without using bins when computing the histogram of the runlengths. For the images in our database, the histograms presented long tails of few elements, especially in the case of Hilbert scannings, so the size of these bins was chosen to increase exponentially. In the experiments, we chose the 25 first bins to have size one, the next 25 to have size two, and for every 25 bins from then on, the size was doubled. When using

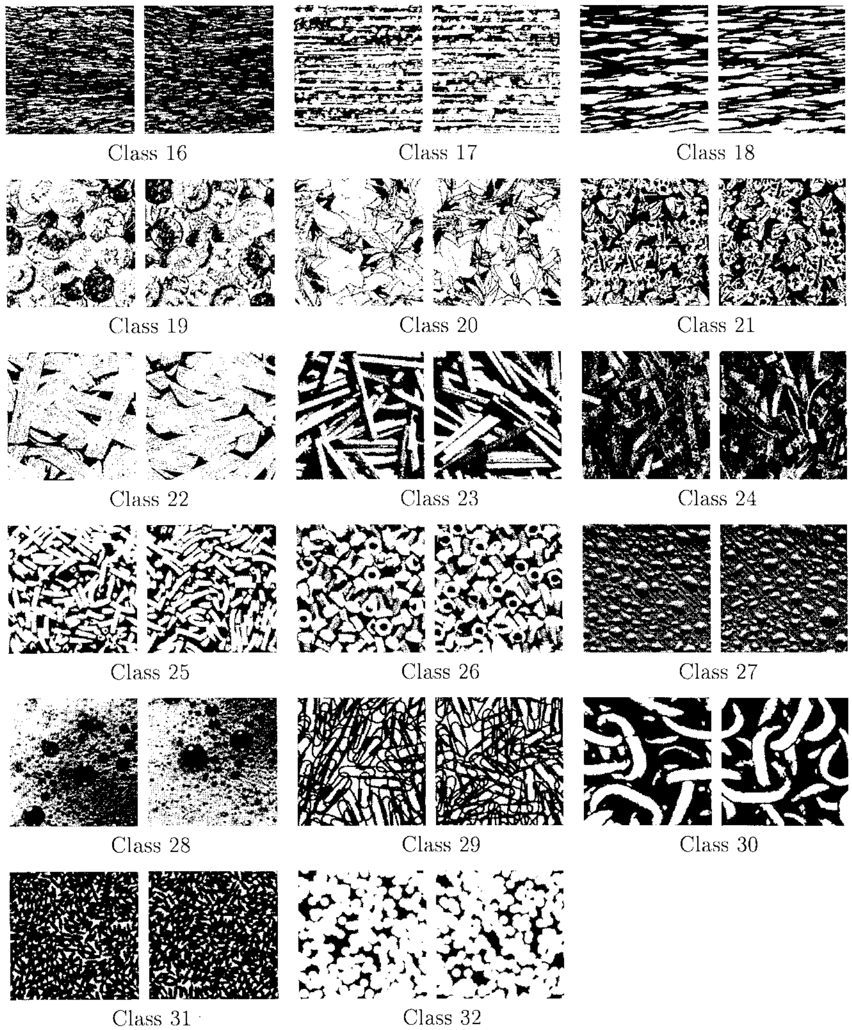


Fig. 4. Texture classes 16–32.

bins, it was necessary to modify the likelihood equation shown in (2) to use $P(m_1 \leq L \leq m_2)$ instead of $P(L = m)$.

Furthermore, experiments were run with models employing different families of probability distributions as well as different scannings and color assignments. The computation of $P(L = m)$ was truncated to 20 significant figures. The detailed results of these experiments can be found in [2].

V. Discussion and Conclusions

From the experiments conducted the following conclusions were drawn.

- It seems that 20 significant figures are enough to characterize a texture image correctly. We also tried with higher number of significant figures and even without truncating the computation of $P(L = m)$, and obtained no important variation in the results. Only in the case of Hilbert scannings we found slight differences in the values of the parameters, but this did not improve the classification accuracy.
- The probability distributions that are defined only by one parameter, that is, Poison, Rayleigh, and Maxwell, do not seem adequate for describing real textures. In all cases they provided worse results (typically in the range 40–85% classification accuracy) than the distributions with two parameters, that is, Normal, Gamma, and Beta which produced classification accuracies typically in the range 60–100%. It is clear that when using probability distributions with two parameters, we can fit better the probability distribution of a real image. Fig. 5 shows the runlength histograms for several images and the probability density functions $P(L = m)$ that provided the maximum likelihood for each family of probability distributions. Note that, in all cases, the probability distributions defined with two parameters provided a function that fitted better the original histogram than the probability functions defined with one parameter.
  The $\mathrm{DN1}(x)$ probability distribution is the one that provided the best results. It performed better in general than the other distributions for all scannings, color assignments, and classification criteria. Furthermore, it was the only one that produced a 100% classification accuracy.
  Using all the probability distributions in the database did not improve the results significantly. In this case, over all the images, scannings and color assignments, the $\mathrm{DN1}(x)$ probability distribution provided the maximum likelihood estimation in 59.4% of the models, the $\mathrm{DN2}(x)$ probability distribution in 4.6%, the Gamma distribution in 10.9%, and the Beta distribution in 25.1%. The Poison, Rayleigh, and Maxwell distributions never provided the maximum likelihood in the estimation.
- The color assignment is a random choice. It was clear from the results that using both possibilities is better than considering only one of them.

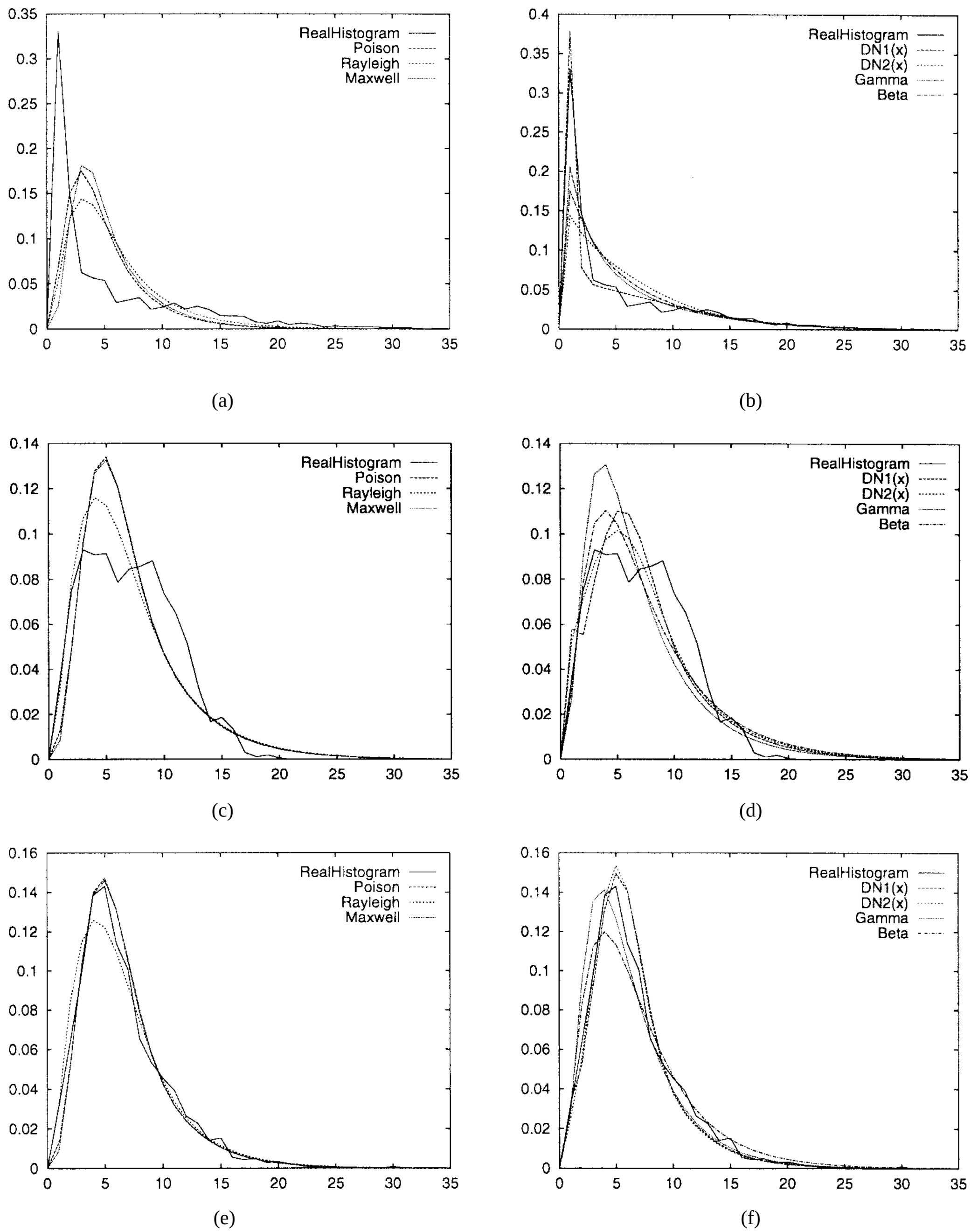
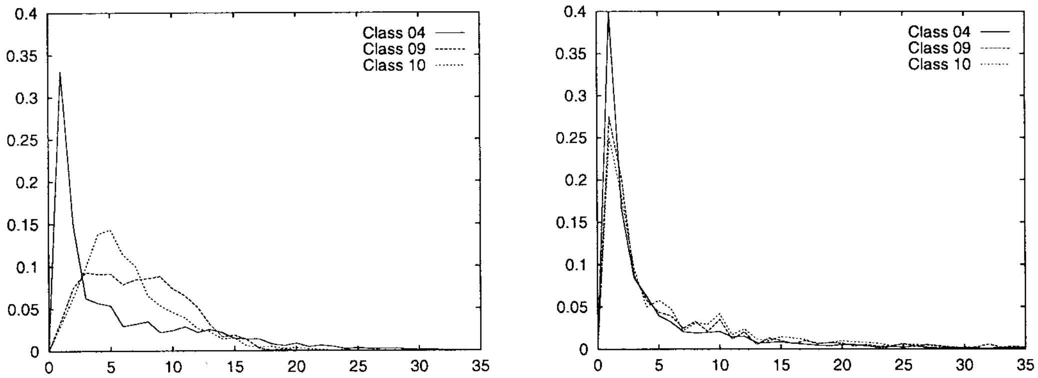


Fig. 5. White runlength histograms for horizontal scanning and $P(L = m)$ functions that provided the maximum likelihood for different families of distribution functions: (a), (b) Test image number four. (c), (d) Test image number nine. (e), (f) Test image number ten. (a), (c), (e) Distributions defined with one parameter. (b), (d), (f) Distributions defined with two parameters.

- When we only use one scanning, the results are worse than when several scannings are considered simultaneously. Obviously, we can find two images that look very different but with the same Boolean model parameters in one direction. Considering more than one directions (several scannings) simultaneously gives a better description of an image.
- The results when using the Hilbert scannings were quite disappointing. We expected that they would provide results similar or even better than the horizontal and vertical scannings, but this did not happen. We tried using different scales for bins and higher levels of accuracy in the computation of $P(L = m)$ but this was not enough to get results similar to the ones obtained with the horizontal and vertical scannings. The Hilbert scannings provide longer runlengths than the horizontal and vertical scannings. This leads to sparsely populated histograms. Furthermore, the shape of the histograms obtained using Hilbert scannings are quite similar even for very different images, while the histograms obtained using horizontal and vertical histograms can be easily distinguished. Fig. 6 shows the white runlength histograms for the test images of classes four, nine, and ten obtained using horizontal scanning (left) and Hilbert scanning (right). Note that, although these images look very different, their corresponding histograms using Hilbert scanning are quite similar. However, the shape of the histograms obtained using the horizontal scanning are sufficiently different to distinguish each class from the others.



(a) (b)

Fig. 6. White runlength histograms (a) using horizontal scanning and (b) using Hilbert scanning.

- The performance of the different classification criteria used is quite similar in most cases. However, the MAXMIN criterion seemed better, and it was the only one that allowed us to obtain a 100% classification accuracy.
- Combining the horizontal and vertical scannings, both possible color assignments and the discrete Normal distributions provided quite good classification results. The correct classification rate is more than 90% with all classification criteria, and a 100% is obtained when using the MAXMIN criterion.

In general:

- The 1-D Boolean model in conjunction with various wrapping formats can produce a diversity of very realistic looking random textures [2].
- The parameters that characterize a particular Boolean model may be estimated with a mean error of 10% and standard deviation of this error of 10% with as few as 512 samples from the data.
- Real textures are not necessarily instantiations of Boolean random processes, but our experiments showed that they can be sufficiently characterized by modeling them as such.
- The proposed method for the classification of binary textures comprising the discretized normal distribution for the shape process and the MAXMIN criterion for classification is a very promising approach, having produced 100% classification accuracy over 32 different textures, requiring less than 4 s to classify an image.

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Regularization of Optic Flow Estimates by Means of Weighted Vector Median Filtering

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***Abstract*— Vector median filtering has been recently proposed as an effective method to refine estimated velocity fields. Here, the use of a *weighted* vector median filtering is suggested to improve the regularization of the optic flow field across motion boundaries. Information about the confidence of the estimated pixel velocities is exploited for the choice of the filter weights. Experimental results, on both synthetic and real-world sequences, show the effectiveness of the proposed procedure.**

***Index Terms*— Optic flow, vector median, vectorfield postprocessing, weighted median.**

I. Introduction

Optic flow is the term to indicate the velocity field generated by the relative motion between the objects and the camera in a framed

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